Lesson 53  More on Evaluating Scientific Formulas

Review:  Shormann Algebra 2, Lessons 4, 5, 12, 16; Shormann Algebra 1, Lesson 60

No New Rules or Definitions

You’ve already done some evaluating of scientific formulas, especially in Lesson 16, plus lots of other evaluate type problems. So, you know by now that all you need to do to solve these is match the variables with the numerical values given in the problem, substitute, and solve.

Within evaluate problems lies a tremendous application of mathematics to the scientific investigation of God’s creation. Scientific formulas don’t ever perfectly explain natural patterns, but sometimes they come incredibly close. Scientific formulas have some incredibly helpful uses, but they can also be misused. Of course, misuse of scientific tools is something we should always be careful to avoid.

As you solve the following problems, don’t be intimidated by strange symbols and units that you may not have seen before. Just focus on answering the question being asked. In these problems, you may need to rearrange first before you solve. We will focus on three main function types, first degree (x), second degree (x^2), and exponential (e^x).

Example 53.1  The potential energy, in Joules, of an object is described by the formula $P.E. = mgh$, where $m =$ mass in kg, $g =$ acceleration due to gravity in m/s^2, and $h =$ altitude in meters. Find the mass of a boulder resting 400 m above sea level, if its potential energy equals $1.2 \times 10^7$ Joules. Assume $g = 10$ m/s^2. Write answer in scientific notation format, rounded to 1 d.p.

solution:  First, rearrange the formula and solve for $m$. Then evaluate. Also, don’t worry about the meaning of all the units, bogging yourself down with questions like “why does potential energy have units of Joules, where did word come from?!” Don’t worry about that right now, that’s for you to learn in your physics class. Instead, focus on the algebra, which means your main questions should be “What am I solving for?” “Did I rearrange the formula correctly?” “Did I substitute the numbers in the right places?” “Does my answer have the right units?”

$$P.E. = mgh$$

$$m = \frac{P.E.}{gh} = \frac{1.2 \times 10^7}{(10)(400)} = 3000 = 3.0 \times 10^3 \text{ kg}$$
Example 53.2 The kinetic energy, in Joules, of an object is described by the formula 
\[ K.E. = \frac{1}{2}mv^2, \] where \( m \) = mass in kg, and \( v \) = the magnitude of the velocity in m/s. Find the velocity of a 500 kg object, if its kinetic energy equals \( 1.2 \times 10^7 \) Joules. Write answer in scientific notation format, rounded to 1 d.p.

solution: Notice the key word “magnitude,” which is often used to refer to a vector’s size, and is normally a positive value. So, when you take the square root of both sides to solve for \( v \), you just need to consider the positive value.

\[
2KE = mv^2 \\
v^2 = \frac{2KE}{m} \\
\sqrt{v^2} = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(1.2 \times 10^7)}{500}} \\
v = 219.09 = 2.2 \times 10^2 \text{ m/s}
\]

Example 53.3 Horizontal position in meters as a function of time is often described using the following function: \( x(t) = v_0 t + 0.5at^2 \). If \( x(2) = 50 \) meters and the initial speed, \( (v_0) = 20 \) m/s, find the acceleration at this instant. Round to 1 d.p. Units for acceleration are m/s\(^2\).

solution: Since this is a function of the form \( x(t) \), the 2 in \( x(2) = 50 \) is the time of the “instant” referred to. Rearrange and solve for \( a \):

\[
x(t) - v_0 t = 0.5at^2 \\
a = \frac{x(t) - v_0 t}{0.5t^2} = \frac{50 - 20(2)}{0.5(2)^2} = \\
\frac{50 - 40}{0.5} = \frac{10}{2} = 5.0 \text{ m/s}^2
\]

Example 53.4 A circuit containing a resistor and capacitor is known as an RC circuit. As a capacitor charges, its current, \( I(t) \) through the resistor decreases exponentially as a function of time in seconds such that \( I(t) = I_0 e^{t/RC} \). For an RC circuit where \( R = 2 \times 10^5 \) Ohms and \( C = 1 \times 10^{-6} \) Farads, find \( I(4) \) when the initial current is 10 amps. Write answer in scientific notation, rounded to 1 d.p.

solution: There are a lot of words in this problem, so be observant and just evaluate \( I(4) \):

\[
I(4) = 10e^{-4/2 \times 10^{-5}} = 2.1 \times 10^{-8} \text{ amps}
\]
Example 53.5 The following is a formula for calculating the future worth, \( F \), of a sum, \( A \), deposited at the end of each year for \( n \) years at an interest rate \( i \).

\[
F = A\left(\frac{(1+i)^n - 1}{i}\right)
\]

Which of the following formulas represents \( A \)?

A) \( \frac{Fi}{(1+i)^n} - 1 \)  
B) \( \frac{Fi}{(1+i)^n} + 1 \)  
C) \( \frac{Fi}{(1+i)^n - 1} \)  
D) \( \frac{F}{i((1+i)^n + 1)} \)

solution: All you need to do is rearrange and solve for \( A \). You don’t need to do anything complicated, just think of the \((1+i)^n - 1\) as one “bag of rocks”:

\[
F = \frac{A((1+i)^n - 1)}{i}
\]

\[
Fi = \frac{F}{i((1+i)^n + 1)}
\]

Therefore, Choice C is correct.

Practice Set 53

(subscripts tell you which lesson each problem came from)

Use your best judgment as to when you should use a calculator. Use 3.14 for \( \pi \).

1. 53' (Medical) The flow rate of blood in \( \text{cm}^3/\text{s} \) is described by the formula \( Q = VA \), where \( V \) is the speed in \( \text{cm}/\text{s} \) and \( A \) is the cross-sectional area of a blood vessel in \( \text{cm}^2 \). Assuming the aorta is circular (so \( A = \pi r^2 \)), with a radius of 1 cm, find the flow rate in a human whose blood flow speed is \( V = 30 \text{ cm/s} \). Round to 1 d.p.

2. 53' The elastic potential energy, in Joules, of a spring is described by the formula \( PE = \frac{1}{2}kx^2 \), where \( k \) is the spring constant in Newtons/m, and \( x \) is the distance the spring is stretched relative to its resting position when \( PE = 0 \). Find the distance a spring of \( K = 1000 \text{ N/m} \) must be stretched to achieve a potential energy of 30 J. Write answer in scientific notation, rounded to 1 d.p.

3. 53' (Electrical engineering) A circuit containing a resistor and capacitor is known as an RC circuit. As a capacitor charges, the voltage across the capacitor decreases exponentially as a function of time in seconds such that \( V(t) = V_0e^{-t/RC} \). For an RC circuit where \( R = 3 \times 10^5 \text{ Ohms} \) and \( C = 1 \times 10^{-6} \text{ Farads} \), find \( V(2) \) when the initial voltage is 12 volts. Write answer in scientific notation, rounded to 1 d.p.

4. 52' If \( x = 3 + i \) and \( y = 2i - 5 \), then \( \frac{x}{y} \) equals?

5. 52' Albert bought 21 resistors, capacitors and LEDs for $30. Find the quantity of each type if resistors cost $1, capacitors cost $2, LEDs cost $3, and there were 3 more capacitors than LEDs.
6. If \( a + bi = (4 + i^3)(3-2i) \), then \( b = ? \)

7. (Medical) Cobalt-60 is a radioactive isotope used to treat cancer. Assuming it decays at an exponential rate, determine the rate constant, \( k \), for Cobalt-60 if its half life is 5.27 years. Round to 3 d.p.

8. (Medical) Barium sulfate (\( \text{BaSO}_4 \)) is a compound administered to patients to get a better x-ray or CT scan of their gastrointestinal tract. What is the percent composition by mass of barium in \( \text{BaSO}_4 \)? Round to 1 d.p. (Ba = 137, S = 32, O = 16).

9. Write the converse, inverse, and contrapositive of the following statement.

   \[ \text{If the shape is a triangle, then the sum of the interior angles equals } 180^\circ. \]

10. Solve. \( \sqrt{x + 4} - 7 = 3 \)

11. Solve the following system.

\[
\begin{align*}
z &= 6x \\
3y + z &= 6 \\
x - 4y - 3z &= 1
\end{align*}
\]

12. Convert the following from polar to rectangular coordinates. \( 9\angle310^\circ \)

13. Find the total surface area of the right, regular pentagonal prism. Round to 1 d.p.

14. A bus starts from Anchorage at noon and heads for Fairbanks, 360 miles away. 2 hours later, a bus leaves Fairbanks and heads for Anchorage at 65 mph. If the buses meet at 4:00 pm, what is the rate of the bus bound for Fairbanks?

15. (CLEP College Algebra) Use the graph of \( h(t) \) to find \( h(h(2)) \).
16. Estimate the x-location of any global extrema on the graph shown.

17. Find the equation of a line that passes through the point (1,0) and is perpendicular to y = 2x - 3.

18. (Medical) The patient’s blood glucose levels followed an exponential relationship, starting at 90 mg/dL, and reaching 151 mg/dL after 30 minutes. Which of the following functions best represents the patient’s glucose levels as function of time?
   A) G(t) = 90 + 0.013t
   B) G(t) = 90 - 241(e^{-0.0453t} - e^{-0.0224t})
   C) G(t) = 151 - .0453t
   D) G(t) = |90 - 0.0224t^3|

19. (Medical) The following is a graph of a patient’s blood glucose level as a function of time. Use the graph to estimate their blood glucose level after t = 250 minutes.

20. Use the intersecting secant angles theorem to find x°. Round to 1 d.p.

\[ m \widehat{AB} = 97.51° \]
\[ m \widehat{BC} = 141.31° \]