Lesson 36  Cartesian Product; More on Sets and Problem Solving

Review:  Shormann Algebra 2, Lessons 12, 26, 27

Rule

**Cartesian Product:** Given two sets, \{A\} and \{B\}, the cartesian product (cross product), A × B, is the set of all ordered pairs (a,b) such that a is a member of \{A\} and b is a member of \{B\}. Using set-builder notation:

\[ A \times B = \{(a,b) \mid a \in A, b \in B\} \]

No new Definitions

36A Cartesian Product

When you see the word *cartesian* in math, that normally means you need to start thinking about the Cartesian plane, or x-y axis. *Cartesian product, or cross product,* is basically the application of sets to analytical geometry and the x-y axis. Cartesian product involves two sets, with \{A\} normally representing the set of inputs, and \{B\} the set of outputs. Cross products have important applications when working with matrices (Lesson 25) and computer programming. In Shormann Algebra 2, we will just focus on the basics of finding cross products when given two sets.

**Example 36.1** Find the Cartesian product, A × B, if A = \{1, 2, 3\} and B = \{7, 8\}.

**solution:** The Cartesian product is the set of all ordered pairs, (x,y), where all x-values come from set A, and all y-values come from set B. Finding the solution is just a matter of writing down all the possible (x,y) pairs. Start with the first element in A, and pair it to the first element in B, and then the second element. Then, repeat the process, this time with the 2nd element of A:

\[ A \times B = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8)\} \]

**Example 36.2** Find the Cartesian product, C × D, if C = \{x, y, z\} and D = \{2, x, -1\}

**solution:** It doesn’t matter if the same element appears in both sets (like the x does). Just find the solution like you did in Ex. 36.1:
$C \times D = \{(x,2), (x,x), (x,-1), (y,2), (y,x), (y,-1), (z,2), (z,x), (z,-1)\}$

**36B More on Sets and Problem Solving**

This section is designed to give you more practice working with sets and the logic behind them. You may want to review Lessons 26 and 27, especially the set notation Rules found in Lesson 26. Set problems are usually pretty easy IF you know the rules!

**Example 36.3** If $A = \{x,y\}$ and $B = \{x,z\}$, which of the following ordered pairs is NOT in the Cartesian product, $A \times B$?

A) $(y,z)$  B) $(x,z)$  C) $(y,y)$  D) $(x,x)$

**solution:** The best thing to do is find the Cartesian product, and then determine which choice is NOT part of it:

$A \times B = \{(x,x), (x,z), (y,x), (y,z)\}$

**Choice C**, $(y,y)$ is not in $A \times B$

**Example 36.4** The data points on the graph represent the Cartesian product $A \times B$. If Set $A = \{1,3\}$, which of the following is set $B$?

A) $\{1,5\}$  B) $\{1\}$  C) $\{2,5\}$  D) $\{5\}$

**solution:** Write down the coordinates of the data points. Then, you should be able to tell which choice is set $B$:

$(1,1), (3,1), (1,5), (3,5)$

Notice that the only values for the $y$, or $B$ coordinate are 1 and 5, which means **Choice A** is correct.

**Example 36.5** If $M = \{x \mid x \geq 0\}$ and $N = \{x \mid x < 2\}$, what is the number of integers in $M \cap N$?

A) one  B) two  C) three  D) four

**solution:** A key word in the problem is *integer*. Start by writing down a few integer values from each set, and see if any numbers are the same, which means they intersect:

$M = 0, 1, 2, 3, 4$  $N = 1, 0, -1, -2$

The sets have two integers in common, 0 and 1, so **Choice B** is correct.
Use your best judgement as to when you should use a calculator. Use 3.14 for $\pi$.

1. Find the Cartesian product of $A \times B$, if $A = \{2,5\}$ and $B = \{1,2,3\}$.

2. If $A = \{a,b\}$ and $B = \{a,c\}$, which of the following ordered pairs is NOT in the Cartesian product, $A \times B$?
   - A) $(a,a)$
   - B) $(a,c)$
   - C) $(a,b)$
   - D) $(b,c)$

3. (Accounting) The company sold t-shirts for $15 and caps for $10. During a week, they sold 800 shirts and caps, totaling $11,250 in sales. Solving which of the following systems yields the number of shirts (s) and caps (c) sold that week?
   - A) $s + c = 800$
   - B) $25(s + c) = 800$
   - $15s + 10c = 11,250$
   - C) $s = 800 - c$
   - D) $15s + c = 800$

4. Find the zeros of $g(x) = x^2 + 5x - 10$

5. If $(ax + 1)(bx + 5) = 12x^2 + cx + 5$ for all values of $x$, and $a + b = 7$, what are the two possible values for $c$? Hint: Expand the left side first.
   - A) 3 and 4
   - B) 16 and -9
   - C) 19 and 23
   - D) 24 and 60

6. If $f(x) = 6x - 1$ and $g(x) = |x - 1|$, find $(g \circ f)(x)$.

7. If $f(x) = \sqrt{3x}$, find $f^{-1}(6)$.

8. If $x#y = x^2 - 2y^2$, find $-2#5$.

9. Is $f(x) = 2x$ an even or odd function?

10. At noon, a bus leaves the station and heads north at 60 mph. Two hours later, another bus leaves the same station and heads south. If the buses are 700 miles apart at 8 pm, how fast is the southbound bus traveling? Round to 1 d.p.

11. The chemical formula for anhydrous gypsum is CaSO$_4$. If the mass of calcium in a block of gypsum weighs 200 g, how much does the oxygen weigh? Round to the nearest gram. ($Ca = 40, S = 32, O = 16$).

12. In the Venn diagram shown, which pair of sets are disjoint?
13. Given \( S = \{1,3,5,7,8\} \), \( T = \{2,5,6,8,10,12\} \), find \( S \cap T \).

14. Mendel crossed a female purebred yellow pea plant (YY) with a homozygous green male plant (yy). Use a Punnett Square to find the probable genotypes of the offspring. Write the probabilities as percentages.

15. (CLEP College Math) The difference between the mean and median of the numbers 10, 11, 11, 14, 24 is
   \[ A) 0 \quad B) 3 \quad C) 4 \quad D) 6 \]

16. If \( x + 2 = 7 \), which rule would you apply to solve for \( x \)?
   A) Additive property of equality
   B) Multiplicative property of equality
   C) Product rule for exponents
   D) Quotient rule for exponents

17. Morgan bought 8 rolls of paper towel for $8.79. At a different store, Chelsea bought 12 rolls of paper towel for $13.29. Who got the better deal?

18. Simplify. \( \frac{x^7}{y} \)

19. Simplify. \( \sqrt{448} \)

20. Which inequality best represents the graph?
   \[ A) \{x \in \mathbb{Z} \mid x > 3\} \quad B) \{x \in \mathbb{Z} \mid x > 4\} \quad C) \{x \in \mathbb{R} \mid x \geq 3\} \quad D) \{x \in \mathbb{R} \mid x \geq 4\} \]